

國立臺中教育大學 112 學年度學士班日間部轉學生招生考試

微積分試題

【本考科得以鉛筆作答】

適用學系：數學教育學系二年級

一、填充題（每題 4%，共 84%）

1. Find $\lim_{x \rightarrow 1} \cos\left(\frac{x^2 - 1}{x - 1}\right) = \underline{\hspace{2cm}}$.

2. Evaluate $\frac{d}{dx} \left[\ln(x^2 + 1) \right] = \underline{\hspace{2cm}}$.

3. Find $\int_0^{\ln 3} e^x \sqrt{1 + e^x} dx = \underline{\hspace{2cm}}$.

4. Evaluate $\frac{d}{dx} \left[\int_1^{\sin x} (1 - t^2) dt \right] = \underline{\hspace{2cm}}$.

5. Calculate $\int_1^{+\infty} \frac{1}{x^3} dx = \underline{\hspace{2cm}}$.

6. Find $\sum_{k=1}^{\infty} \left(\frac{3}{4^k} - \frac{2}{5^{k-1}} \right) = \underline{\hspace{2cm}}$.

7. The slope of the tangent line to the curve $y^2 - x + 1 = 0$ at the point $(4, 3)$ is $\underline{\hspace{2cm}}$.

8. Evaluate $\lim_{x \rightarrow 1} \sin^{-1} \left(\frac{1 - \sqrt{x}}{x - 1} \right) = \underline{\hspace{2cm}}$.

9. At what point on the curve $y = 2 + 3e^x - 4x$ is the tangent line parallel to the line $5x - y = 6$? $\underline{\hspace{2cm}}$

（背面尚有試題）

10. Let $f(x) = e^{x^2} + x^{e^2} + x^{\ln x} + e^2$. Then $f'(x) =$ _____.

11. The points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$ are _____.

12. If $f(x) = \int_0^{x^2} (1-t)e^{t^2} dt$, on what intervals is f increasing? _____

13. The area enclosed by the line $y = \frac{1}{2}x$ and the parabola $x = y^2 - 4y$ is _____.

14. Evaluate $\int e^{\sqrt{x}} dx =$ _____.

15. Evaluate $\int_0^1 x^3 e^{-x^2} dx =$ _____.

16. Evaluate $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n (\ln(n+k) - \ln(n)) =$ _____.

17. If $f(x) = 10^{\tan x}$, then $f'(\frac{\pi}{4}) =$ _____.

18. Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx =$ _____.

19. Evaluate $\lim_{x \rightarrow 0} \left(\frac{3^x + 5^x}{2} \right)^{1/x} =$ _____.

20. Given that $f(x) = 2x + \cos x$, find $(f^{-1})'(1) =$ _____.

21. Suppose f is continuous on $(0, \infty)$ and, $\int_0^{x^2} f(t) dt = 1 + x$ for $x > 0$. Then $f(4) =$ _____.

二、計算證明題（每題 8%，共 16%）

1. Find the area of the region that is inside of the cardioid $r = 4 + 4\cos \theta$ and outside of the circle $r = 6$.

2. Find a, b such that $\lim_{x \rightarrow 1} \frac{\sqrt[3]{ax + b} - 2}{x - 1} = 1$.